GATE ACADEMY Presents

Most Awaited Book For

GATE - 2020

Mechanical Engineering
Best Book for GATE Exam

GATE

2020

MECHANICAL ENGINEERING

Volume - I

- Meticulous Solution of GATE Previous Year Questions (1987-2019)
- Multi Method Approach for a Single Problem to Develop Crystal Clear Concepts
- This book is also a value addition for ESE/PSUs/ISRO/DRDO

GATE ACADEMY PUBLICATIONS
...The Mentor for Engineers
<table>
<thead>
<tr>
<th>S. No.</th>
<th>Topics</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Fluid Mechanics</td>
<td>1.1 - 1.000</td>
</tr>
<tr>
<td>1.</td>
<td>Properties of Fluid</td>
<td>1.1 - 1.00</td>
</tr>
<tr>
<td>2.</td>
<td>Pressure &amp; Its Measurement</td>
<td>1.00 - 1.00</td>
</tr>
<tr>
<td>3.</td>
<td>Hydrostatic Forces</td>
<td>1.00 - 1.00</td>
</tr>
<tr>
<td>4.</td>
<td>Buoyancy &amp; Floatation</td>
<td>1.00 - 1.00</td>
</tr>
<tr>
<td>5.</td>
<td>Kinematics of Fluid</td>
<td>1.00 - 1.00</td>
</tr>
<tr>
<td>6.</td>
<td>Dynamics of Fluid</td>
<td>1.00 - 1.00</td>
</tr>
<tr>
<td>7.</td>
<td>Laminar &amp; Turbulent Flow, Viscous Flow, Flow Through Pipes</td>
<td>1.00 - 1.00</td>
</tr>
<tr>
<td>8.</td>
<td>Boundary Layer Theory</td>
<td>1.00 - 1.00</td>
</tr>
<tr>
<td>9.</td>
<td>Hydraulic Machines</td>
<td>1.00 - 1.00</td>
</tr>
<tr>
<td>2.</td>
<td>Thermodynamics</td>
<td>2.1 - 2.000</td>
</tr>
<tr>
<td>1.</td>
<td>Thermodynamics System &amp; Process</td>
<td>2.1 - 2.00</td>
</tr>
<tr>
<td>2.</td>
<td>First Law of Thermodynamics</td>
<td>2.00 - 2.00</td>
</tr>
<tr>
<td>3.</td>
<td>Second Law of Thermodynamics</td>
<td>2.00 - 2.00</td>
</tr>
<tr>
<td>4.</td>
<td>Entropy</td>
<td>2.00 - 2.00</td>
</tr>
<tr>
<td>5.</td>
<td>Availability &amp; Irreversibility</td>
<td>2.00 - 2.00</td>
</tr>
<tr>
<td>6.</td>
<td>Pure Substances</td>
<td>2.00 - 2.00</td>
</tr>
<tr>
<td>7.</td>
<td>Steam Power Cycles</td>
<td>2.00 - 2.00</td>
</tr>
<tr>
<td>8.</td>
<td>Gas Power Cycles</td>
<td>2.00 - 2.00</td>
</tr>
<tr>
<td>9.</td>
<td>Internal Combustion Engine</td>
<td>2.00 - 2.00</td>
</tr>
<tr>
<td>3.</td>
<td>Heat Transfer</td>
<td>3.1 - 3.000</td>
</tr>
<tr>
<td>1.</td>
<td>Conduction</td>
<td>3.1 - 3.0</td>
</tr>
<tr>
<td>2.</td>
<td>Fins &amp; Unsteady Heat Transfer</td>
<td>3.0 - 3.00</td>
</tr>
<tr>
<td>3.</td>
<td>Free &amp; Forced Convection</td>
<td>3.00 - 3.00</td>
</tr>
<tr>
<td>4.</td>
<td>Heat Exchanger</td>
<td>3.00 - 3.00</td>
</tr>
<tr>
<td>5.</td>
<td>Radiation</td>
<td>3.00 - 3.00</td>
</tr>
</tbody>
</table>
### 4. Refrigeration & Air-Conditioning

<table>
<thead>
<tr>
<th></th>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vapour Refrigeration</td>
<td>4.1 - 4.0</td>
</tr>
<tr>
<td>2</td>
<td>Properties of Moist Air &amp; Psychrometric Processes</td>
<td>4.0 - 4.00</td>
</tr>
<tr>
<td>3</td>
<td>Heat Pumps &amp; Cycles</td>
<td>4.00 - 4.00</td>
</tr>
</tbody>
</table>

### 5. Mechanics of Solids

<table>
<thead>
<tr>
<th></th>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Stress &amp; Strain</td>
<td>5.1 - 5.0</td>
</tr>
<tr>
<td>2</td>
<td>Complex Stress</td>
<td>5.0 - 5.00</td>
</tr>
<tr>
<td>3</td>
<td>Elastic Constants &amp; Theory of Failure</td>
<td>5.00 - 5.00</td>
</tr>
<tr>
<td>4</td>
<td>Thin Cylinder</td>
<td>5.00 - 5.00</td>
</tr>
<tr>
<td>5</td>
<td>Shear Force &amp; Bending Moment Diagrams</td>
<td>5.00 - 5.00</td>
</tr>
<tr>
<td>6</td>
<td>Bending of Beam</td>
<td>5.00 - 5.00</td>
</tr>
<tr>
<td>7</td>
<td>Torsion of Shaft</td>
<td>5.00 - 5.00</td>
</tr>
<tr>
<td>8</td>
<td>Springs</td>
<td>5.00 - 5.00</td>
</tr>
<tr>
<td>9</td>
<td>Euler’s Theory of Column</td>
<td>5.00 - 5.00</td>
</tr>
<tr>
<td>10</td>
<td>Strain Energy &amp; Thermal Stresses</td>
<td>5.00 - 5.00</td>
</tr>
</tbody>
</table>

### 6. Machine Design

<table>
<thead>
<tr>
<th></th>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Design Against Static Load (Theory of Failure)</td>
<td>6.1 - 6.0</td>
</tr>
<tr>
<td>2</td>
<td>Design Against Dynamic Load (Fatigue Strength &amp; SN Diagram)</td>
<td>6.0 - 6.00</td>
</tr>
<tr>
<td>3</td>
<td>Gears</td>
<td>6.00 - 6.00</td>
</tr>
<tr>
<td>4</td>
<td>Bearings, Shaft &amp; Keys</td>
<td>6.00 - 6.00</td>
</tr>
<tr>
<td>5</td>
<td>Clutches, Ropes &amp; Belts</td>
<td>6.00 - 6.00</td>
</tr>
<tr>
<td>6</td>
<td>Brakes</td>
<td>6.00 - 6.00</td>
</tr>
<tr>
<td>7</td>
<td>Joints (Bolted, Riveted &amp; Welded)</td>
<td>6.00 - 6.00</td>
</tr>
<tr>
<td>8</td>
<td>Power Screws &amp; Springs</td>
<td>6.00 - 6.00</td>
</tr>
</tbody>
</table>

### 7. General Aptitude

<table>
<thead>
<tr>
<th></th>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Numerical Ability</td>
<td>7.1 – 7.0</td>
</tr>
<tr>
<td>2</td>
<td>Logical Reasoning</td>
<td>7.0 - 7.00</td>
</tr>
<tr>
<td>3</td>
<td>Verbal Ability</td>
<td>7.00 - 7.00</td>
</tr>
</tbody>
</table>
1.1 A fluid is said to be Newtonian fluid when the shear stress is 
(A) Directly proportional to the velocity gradient. 
(B) Inversely proportional to the velocity gradient. 
(C) Independent of the velocity gradient. 
(D) None of the above.

1.2 A fluid is one which can be defined as a substance that 
(A) Has that same shear stress at all points. 
(B) Can deform indefinitely under the action of the smallest shear force. 
(C) Has the small shear stress in all directions. 
(D) Is practically incompressible.

1.3 The dimension of surface tension is 
(A) ML⁻¹ 
(B) L²T⁻¹ 
(C) ML⁻¹T⁻¹ 
(D) MT⁻²

1.4 The dimension of surface tension is 
(A) N/m² 
(B) J/m 
(C) J/m² 
(D) W/m

1.5 Match 4 correct pairs between List-I and List-II. 

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Steam nozzle</td>
<td>1. Mach Number</td>
</tr>
<tr>
<td>B. Compressible Flow</td>
<td>2. Reaction Turbine</td>
</tr>
<tr>
<td>C. Surface tension</td>
<td>3. Biot Number</td>
</tr>
<tr>
<td>D. Heat conduction</td>
<td>4. Nusselt Number</td>
</tr>
<tr>
<td></td>
<td>5. Super saturation</td>
</tr>
<tr>
<td></td>
<td>6. Weber Number</td>
</tr>
</tbody>
</table>

1.6 If 'p' is the gauge pressure within a spherical droplet, then gauge pressure within a bubble of the same fluid and of same size will be 
(A) p / 4 
(B) p / 2 
(C) p 
(D) 2p

1.7 Kinematic viscosity of air at 20°C is given to be 1.6 × 10⁻⁵ m²/s. Its kinematic viscosity at 70°C will be varying approximately: 
(A) 2.2 × 10⁻⁵ m²/s 
(B) 1.6 × 10⁻⁵ m²/s 
(C) 1.2 × 10⁻⁵ m²/s 
(D) 3.2 × 10⁻⁵ m²/s
1.8 The S.I. unit of kinematic viscosity (ν) is, [1 Mark]
(A) m²/sec (B) kg/m-sec
(C) m/sec² (D) m³/sec²

1.9 A static fluid can have [1 Mark]
(A) Non-zero normal and shear stress.
(B) Negative normal stress and zero shear stress.
(C) Positive normal stress and zero shear stress.
(D) Zero normal stress and non-zero shear stress.

1.10 An incompressible fluid (kinematic viscosity 7.4×10⁻⁷ m²/s, specific gravity 0.88) is held between two parallel plates. If the top plate is moved with a velocity of 0.5 m/s while the bottom one is held stationary, the fluid attains a linear velocity profile in the gap of 0.5 mm between these plates the shear stress in pascals on the surface of top plate is [1 Mark]
(A) 0.651×10⁻³ (B) 0.651
(C) 6.51 (D) 0.651×10⁻¹

1.11 A cubic block of side ‘L’ and mass ‘M’ is dragged over an oil film across the table by a string connects to a hanging block of mass ‘m’ as shown is fig. The Newtonian oil film of thickness ‘h’ has dynamic viscosity ‘μ’ and the flow condition is laminar. The acceleration due to gravity is ‘g’. The steady state velocity ‘V’ of block is [2 Marks]

1.12 Newton’s law of viscosity states that the shear stress in a fluid is proportional to [2 Marks]
(A) The velocity of the fluid.
(B) The time rate of change of velocity of the fluid.
(C) The rate of change of velocity of the fluid change with height of fluid film.
(D) The square of the velocity of the fluid.

1.13 For a Newtonian fluid [1 Mark]
(A) Shear stress is proportional to shear strain.
(B) Rate of shear stress is proportional to shear strain.
(C) Shear stress is proportional to rate of shear strain.
(D) Rate of shear stress is proportional to rate of shear strain.

1.14 Oil in a hydraulic cylinder is compressed from an initial volume 2 m³ to 1.96 m³. If the pressure of oil in the cylinder changes from 40 MPa to 80 MPa during compression, the bulk modulus of elasticity of oil is [2 Marks]
(A) 1000 MPa (B) 2000 MPa
(C) 4000 MPa (D) 8000 MPa
1.15 A journal bearing has a shaft diameter of 40 mm and a length of 40 mm. The shaft is rotating at 20 rad/s and the viscosity of the lubricant is 20 mPa-s. The clearance is 0.020 mm. The loss of torque due to the viscosity of the lubricant is approximately \[2 \text{ Marks}\]
(A) 0.040 Nm  (B) 0.252 Nm  (C) 0.400 Nm  (D) 0.652 Nm

1.16 A two dimensional fluid element rotates like a rigid body. At a point within the elements, the pressure is 1 unit. Radius of the Mohr’s circle, characterizing the state of stress at the point is \[2 \text{ Marks}\]
(A) 0.5 unit  (B) 0 unit  (C) 1 unit  (D) 2 units

1.17 A lightly loaded full journal bearing has journal diameter of 50 mm, bush bore of 50.50 mm and bush length of 20 mm. If rotational speed of journal is 1200 rpm and average viscosity of liquid lubricant is 0.3 Pa-sec, the power loss (in Watt) will be \[2 \text{ Marks}\]
(A) 37  (B) 74  (C) 118  (D) 237

1.18 The difference in pressure (in N/m\(^2\)) across an air bubble of diameter 0.001 m immersed in water (surface tension \(= 0.072 \text{ N/m}\)) is ______. \[1 \text{ Mark}\]

1.19 In a simple concentric shaft-bearing arrangement, the lubricant flows in the 2 mm gap between the shaft and the bearing. The flow may be assumed to be a plane Couette flow with zero pressure gradient. The diameter of the shaft is 100 mm and its tangential speed is 10 m/s. The dynamic viscosity of the lubricant is 0.1 kg/ms. The frictional resisting force (in newton) per 100 mm length of the bearing is ______. \[2 \text{ Marks}\]

1.20 Couette flow is characterized by \[1 \text{ Mark}\]
(A) Steady, incompressible, laminar flow through a straight circular pipe.  
(B) Fully developed turbulent flow through a straight circular pipe.  
(C) Steady, incompressible, laminar flow between two fixed parallel plates.  
(D) Steady, incompressible, laminar flow between one fixed plate and the other moving with a constant velocity.

1.21 Consider fluid flow between two infinite horizontal plates which are parallel (the gap between them being 50 mm). The top plate is sliding parallel to the stationary bottom plate at a speed of 3 m/s. The flow between the plates is solely due to the motion of the top plate. The force per unit area (magnitude) required to maintain the bottom plate stationary is _____ 2N/m. Viscosity of the fluid \(\mu = 0.44 \text{ kg/m-s}\) and density \(\rho = 888 \text{ kg/m}^3\). \[2 \text{ Marks}\]
1.22 A solid block of 2.0 kg mass slides steadily at a velocity $V$ along a vertical wall as shown in the figure below. A thin oil film of thickness $h = 0.15$ mm provides lubrication between the block and the wall. The surface area of the face of the block in contact with the oil film is $0.04 \text{ m}^2$. The velocity distribution within the oil film gap is linear as shown in the figure. Take dynamic viscosity of oil as $7 \times 10^{-3}$ Pa·s and acceleration due to gravity as $10 \text{ m/s}^2$. Neglect weight of the oil. The terminal velocity $V$ (in m/s) of the block is ______ (correct to one decimal place) [2 Marks]

![Diagram of a block sliding on a wall with a lubricating oil film.](image)

1.23 Two immiscible, incompressible, viscous fluids having same densities but different viscosities are contained between two infinite horizontal parallel plates, 2 m apart as shown below. The bottom plate is fixed and the upper plate moves to the right with a constant velocity of 3 m/s. With the assumptions of Newtonian fluid, steady and fully developed laminar flow with zero pressure gradient in all directions, the momentum equations simplify to

$$\frac{d^2 u}{dy^2} = 0.$$
**Newtonian fluid:**
- Fluid which obey Newton’s law of viscosity are known as *Newtonian fluids*.
- According to Newton’s law of viscosity shear stress ($\tau$) on a fluid element layer is directly proportional to velocity gradient ($\frac{du}{dy}$).

$$\Rightarrow \quad \tau \propto \frac{du}{dy}$$

Shear stress, $\tau = \mu \frac{du}{dy}$

But in general form, $\tau = \mu \left(\frac{du}{dy}\right)^n$

where, $n = 1$ (For Newtonian fluid)
$n > 1$ (For Dilatant fluid)
$n < 1$ (For Pseudo plastic fluid)

**Examples of Newtonian fluid:**
- Water, Air, Petrol, Kerosene, Diesel, Alcohol and Gasoline.

Hence, the correct option is (A).
1.2 (B)

- A substance in liquid or gaseous phase is referred as fluid. They are capable of deforming continuously under the action of tangential force or shear force, however smallest the shear force and tangential force may be.
- Because of continuous deformation fluid has a ability to flow.

Hence, the correct option is (B).

1.3 (D)

Surface tension is the elastic tendency of a fluid surface which makes it acquire the least surface area possible. Surface tension allows insects (e.g. water striders), usually denser than water, to float and stride on a water surface.

It is denoted by Greek letter \( \sigma \) (sigma). In M.K.S. units, it is expressed as kgf/m while in S.I. units as N/m.

\[
\text{Surface tension} \ (\sigma) = \frac{\text{Tension force} \ (F)}{\text{Length} \ (L)}
\]

\[
\sigma = \frac{\text{MLT}^{-2}}{\text{L}} = \text{MT}^{-2}
\]

Hence, the correct option is (D).

Key Point

Consider a molecule \( A, B \) and \( C \) as shown in figure, net force in particle \( A \) is zero it is in complete equilibrium (stable condition).

Particle \( B \) and \( C \) which are present at the free surface, is not in equilibrium, net force acting downward, because of this net downward force a tensile force is induced as shown in figure, that tensile force per unit length is called surface tension. Denoted by \( \sigma \).

1.4 (C)

Surface tension is defined as surface energy per unit surface area.

\[
\sigma = \frac{\text{Surface energy}}{\text{Surface area}} \left( \frac{J}{m^2} \right)
\]

Surface tension is because of net force which acts downward in the liquid particle which are present at the free surface.

Hence, the correct option is (C).

Key Point

- The surface energy is defined as the energy associated with the intermolecular forces at the interface between two media.
- Another unit of surface tension is N/m .
- Surface tension is responsible because of unbalance cohesive force \( F_c \). This unbalance cohesive force is balance by tension force.
- At the critical temperature surface tension is zero.
Surface tension is a binary property of a liquid and gas or two liquids, depending on the surrounding fluid also.

**Example:**
\[
\sigma_{H_2O-Air} = 0.072 \text{ N/m}
\]
and,
\[
\sigma_{Hg-Air} = 0.48 \text{ N/m}
\]

(A \rightarrow 5) : Supersaturation of steam takes place in steam nozzle due to delay in condensation.
(B \rightarrow 1) : Mach number is related to compressible flow.
(C \rightarrow 6) : Weber number is related to surface tension.
(D \rightarrow 3) : Biot number is relevant to heat conduction.

**1.5 (A-5, B-1, C-6, D-3)**

**1.6 (B)**

**Given :** Gauge pressure within a spherical droplet is \( p \).

**Case 1 :** (Spherical droplet/liquid droplet/water droplet)

Let, \( p = \text{Pressure inside the droplet above outside pressure } (\Delta p = p - p_0 = p) \)
\( D = \text{Diameter of droplet} \)
\( \sigma = \text{Surface tension of the liquid} \).

Busting force = \( \Delta p \times \frac{\pi}{4} D^2 \) \hspace{1cm} \text{(Pressure force)}

and, resisting force = \( \sigma \times \pi D \) \hspace{1cm} \text{(Surface tension force)}

At equilibrium,
Busting force = Resisting force
\[
\Delta p \times \frac{\pi}{4} D^2 = \sigma \pi D + \sigma \pi (D + 2t)
\]
\( ( : D + 2t \approx D) \)
\[
\Delta p \times \frac{\pi}{4} D^2 = 2 \sigma \pi D
\]
\[
p = p_{in} - p_{out} = \Delta p = \frac{8 \sigma}{D} \hspace{1cm} \text{...(ii)}
\]

From equations (i) and (ii), we get
\[
P_{bubble} = 2P_{droplet}
\]

Hence, the correct option is (B).

**Key Point**

Due to thickness of bubble it provides double resisting force (i.e., by both inner and outer film of bubble) compare to liquid droplet. So, to maintain equilibrium the value of pressure of bubble is become two times that of liquid droplet.

**1.7 (A)**

**Given :**

Initial temperature of air \( (T_1) = 20^\circ C = 293 K \)

Initial kinematic viscosity of air \( (\nu_1) = 1.6 \times 10^{-5} \text{ m}^2/\text{s} \)

Find temperature of air \( (T_2) = 70^\circ C = 343 K \)
The dynamic viscosity ($\mu$) of gases increase with *increase in temperature*.  
\[ \mu \propto \sqrt{T} \]
Density of gases decrease with increase in temperature at constant pressure.  
\[ \rho \propto \frac{1}{T} \]
So, kinematic viscosity ($\nu$) is the ratio of dynamic viscosity to density,
\[ \nu = \frac{\mu}{\rho} \]
\[ \nu \propto \frac{\sqrt{T}}{1/T} \propto T^{3/2} \]
\[ \frac{\nu_1}{T^{3/2}} = \text{constant} \]
\[ \frac{\nu_1}{(T_1)^{3/2}} = \frac{\nu_2}{(T_2)^{3/2}} \]
\[ \frac{1.6 \times 10^{-5}}{(293)^{3/2}} = \frac{\nu_2}{(343)^{3/2}} \]
\[ \nu_2 = 2.026 \times 10^{-5} \text{ m}^2/\text{s} \]
Hence, the correct option is (A).

### Key Point

The dynamic viscosity ($\mu$) of gases increase with *increase in temperature*. This is due to the reason that in gases the intermolecular cohesion is negligible and the shear stress is due to exchange of momentum of the molecule, normal to the direction of motion. The molecular activity increases with rise in temperature and so does the viscosity of gas.

**Effect of temperature:**

For gases,  \[ \uparrow \nu_{\text{gas}} = \frac{\mu_{\text{gas}}}{\rho_{\text{gas}}} \uparrow \]  
(when \( T \uparrow \))  
Where,  \( \uparrow = \text{Increases}, \ \downarrow = \text{Decreases}. \)

For liquids,  \[ \downarrow \nu_{\text{liquid}} = \frac{\mu_{\text{liquid}}}{\rho_{\text{liquid}} (c)} \downarrow \]  
(when \( T \uparrow \))  
Where, \( c = \text{constant}. \)
Normal stress at the surface of the fluid element
$$\frac{F_a}{dA} = F \sin \theta \frac{dA}{dA}$$

Normal stress and shear stress are vector quantities.
For a static fluid body, i.e., a body of fluid that is at rest or has zero velocity, the shear stress is always zero. Also for static fluids, the normal stress is always positive.
Hence, the correct option is (C).

**Key Point**

**Real fluid:**

- Real fluid ($\mu \neq 0$)
- Static ($\mu = 0$)
  - $du = 0$
  - $\tau = 0$
  - Pascal law is valid
- Motion ($\mu \neq 0$)
  - $du \neq 0$
  - $\tau = \frac{\mu du}{dy}$
  - Pascal law is not valid

**Ideal fluid:**

- Ideal fluid ($\mu = 0$)
- Static ($\mu = 0$)
  - $du = 0$
  - $\tau = 0$
  - Pascal law is valid
- Motion ($\mu \neq 0$)
  - $du \neq 0$
  - $\tau = 0$
  - Pascal law is valid

**Given:**

- Kinematic viscosity ($\nu$) = $7.4 \times 10^{-7}$ m$^2$/s
- Specific gravity ($s$) = 0.88
- Density ($\rho$) = 0.88 $\times$ 1000 = 880 kg/m$^3$
- Velocity of plate ($V$) = 0.5 m/s

From Newton’s law of viscosity,
$$\tau = \mu \frac{\partial u}{\partial y}$$

As the gap is very small, we can assume that velocity profile is linear. For linear velocity profile,
$$\tau = \frac{\mu V}{y} = \rho u \frac{V}{y} \quad \left(\because \frac{\mu}{\rho}\right)$$
$$\tau = 7.4 \times 10^{-7} \times 880 \times \frac{0.5}{0.5 \times 10^{-3}}$$
$$\tau = 0.651 \text{ Pa}$$
Hence, the correct option is (B).

According question F.B.D. of small block,

Now from F.B.D., $T = mg$

Viscous force, $F = T = mg$

(Acts on the upper layer of oil)

$$\therefore \quad F = \frac{\mu AV}{y} \quad \left(\because \tau = \frac{\mu V}{y} \& F = \tau A\right)$$

$$mg = \frac{\mu AV}{h}$$

$$V = \frac{mgh}{\mu L}$$

Hence, the correct option is (C).
For a Newtonian fluid, shear stress is directly proportional to the velocity gradient or the rate of deformation or the rate of angular displacement or the rate of shear strain.

$$\tau \propto \frac{du}{dy} \quad \text{or} \quad \tau = \mu \frac{du}{dy}$$

Hence, the correct option is (C).

### Key Point

**For solids**: Shear stress depends upon shear force and not on time. So,

$$\tau \propto \phi$$

**For fluids**: But for fluids shear strain ($\phi$) depends on both shear stress ($\tau$) as well as time.

### Given:

Hydraulic cylinder initial volume ($V_1$) = 2 m$^3$

Hydraulic cylinder Final volume ($V_2$) = 1.96 m$^3$

Initial pressure of oil cylinder ($p_1$) = 40 MPa

Final pressure of oil cylinder ($p_2$) = 80 MPa

Volume difference,

$$dV = V_2 - V_1 = 1.96 - 2 = -0.04 \text{ m}^3$$

Pressure difference,

$$dp = p_2 - p_1 = 80 - 40 = 40 \text{ MPa}$$

Bulk modulus of elasticity,

$$K = \left( -\frac{dp}{dV} \right) = \frac{-40 \times 2}{-0.04} = 2000 \text{ MPa}$$

Hence, the correct option is (B).

### Key Point

**Compressibility**: It is defined as the change in volume with respect to change in pressure. The coefficient of compressibility is measure of compressibility.

**Coefficient of compressibility ($\beta$)**:

It is define as the relative change in volume with respect to change pressure.

$$\beta = -\frac{\Delta V}{V} = -\frac{dV}{dp}$$

(–ve sign indicated when pressure increase volume decrease).

$$\beta = \left( -\frac{dV}{V} = \frac{dp}{\rho} \right)$$

Relative change in density,

$$\Rightarrow \frac{d\rho}{\rho} \leq 5\% \quad \text{(Assuming incompressible substance)}$$

$$\Rightarrow \frac{d\rho}{\rho} > 5\% \quad \text{(Compressible substance)}$$

**Example**:

1 atm $\rightarrow \rho_{\text{water}} = 998 \text{ kg/m}^3$

1000 atm $\rightarrow \rho_{\text{water}} = 1003 \text{ kg/m}^3$

Relative change in density

$$= \frac{998 - 1003}{998} \times 100 = 0.5\%$$

(Incompressible)

Theoretically, compressibility of water is approximately zero.

$$\beta_{\text{water}} \approx 0$$

**Bulk modulus or dilation modulus ($K$)**:

Reciprocal of coefficient of compressibility is called bulk modulus or dilation modulus i.e.,

$$K = \frac{1}{\beta}$$

$$(K)_{\text{water}} = \frac{1}{\beta} = \frac{1}{0} = \infty$$
Given:
- Diameter of shaft \((D) = 40\) mm
- Length of shaft \((L) = 40\) mm
- Speed of shaft \((\omega) = 20\) rad/s
- Viscosity of lubricant \((\mu) = 20\) mPa-s = \(20 \times 10^{-3}\) Pa-s
- Clearance \((y) = 0.020\) mm

Torque due to viscosity,

\[F = \mu \times A \times \frac{V}{y}\]

\[A = \pi DL = \pi \times 40 \times 10^{-3} \times 40 \times 10^{-3}\]

\[A = 5.0265 \times 10^{-3}\]

Velocity, \(V = \omega \times r = \omega \times \frac{d}{2}\)

\[V = 20 \times \frac{40 \times 10^{-3}}{2} = 0.4\] m/s

\[F = 20 \times 10^{-3} \times 5.0265 \times 10^{-3} \times \frac{0.4}{0.020 \times 10^{-3}}\]

\[F = 2.0106\] N

\[T = F \times r = F \times \frac{d}{2}\]

\[T = 2.0106 \times \frac{40 \times 10^{-3}}{2} = 0.040\] Nm

Hence the correct option is (A).

Method 1
By graphical approach:

Since Mohr’s circle is a point, so the radius of Mohr’s circle is zero.

Hence, the correct option is (B).

Method 2
By analytical approach:

Stress in \(x\)-direction \((\sigma_x) = -p\)

Stress in \(y\)-direction \((\sigma_y) = -p\)

Radius of Mohr circle is given by,

\[R = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{2} + \tau^2}\]

\[R = \sqrt{\frac{(-p - (-p))^2}{2} + 0}\]

\[R = \sqrt{\frac{(-p + p)^2}{2} + 0} = 0\]

Hence, the correct option is (B).

1.16 (B)

In rigid body motion, velocity is uniform. As velocity is uniform, velocity gradient is zero. i.e., \(\frac{du}{dy} = 0\), as \(\frac{du}{dy} = 0\), shear stress \(\tau = 0\).

If \(\tau = 0\), pressure is uniform in all directions at a point.

Stress element at point is,

\[p = \sigma_x\]

\[p = \sigma_y\]

Given:
- Diameter of bush \((D_b) = 50.5\) mm
- Diameter of journal \((D) = 50\) mm
- Length of bush \((L) = 20\) mm
- Speed of journal \((N) = 1200\) rpm
- Viscosity of lubricant \((\mu) = 0.3\) Pa-s

Torque generated by lubricant,

\[T = F \times r = \mu \times A \times \frac{V}{y} \times \frac{D}{2} \quad \text{...(i)}\]
\[ A = \pi DL \]
\[ A = \pi \times (50 \times 10^{-3}) \times (20 \times 10^{-3}) \]
\[ A = 3.141 \times 10^{-3} \]
\[ V = \frac{\pi DN}{60} = \frac{\pi \times 0.05 \times 1200}{60} = 3.141 \text{ m/s} \]
\[ y = \frac{D_n - D}{2} = \frac{0.0505 - 0.05}{2} = 2.5 \times 10^{-4} \]

Putting all the values in equation (i),
\[ T = 0.3 \times 3.141 \times 10^{-3} \times \frac{3.141}{2.5 \times 10^{-4}} \times 0.05 \]
\[ T = 0.2959 \text{ N-m} \]

Power loss,
\[ P = \frac{2\pi NT}{60} \]
\[ P = \frac{2 \times \pi \times 1200 \times 0.2959}{60} = 37.19 \text{ Watt} \]

Hence the correct option is (A).

**1.18 288**

**Given :** Diameter of air bubble \((D) = 0.001 \text{ m}\)
Surface tension of water \((\sigma) = 0.072 \text{ N/m}\)
Pressure difference in air bubble immersed in water is given by,
\[ \Delta p = \frac{4\sigma}{D} = \frac{4 \times 0.072}{0.001} = 288 \text{ N/m}^2 \]

Hence, the difference in pressure across an air bubble immersed in water is **288 N/m\(^2\)**.

**Key Point**

**Case-1 :**
Two surface or two interface,
\[ \text{Air} \quad \text{Air} \quad \text{Air} \quad \text{Water} \]

It is a bubble, so formula used for calculating pressure difference is,
\[ \Delta p = \frac{8\sigma}{D} \]

**Case-2 :**
One interface or one surface,
\[ \text{Air} \quad \text{Air} \quad \text{Air} \quad \text{Water} \]

Formula used for calculating pressure difference is,
\[ \Delta p = \frac{4\sigma}{D} \]

Since, here only one common surface (interface), that’s why in this case this bubble is acting as a drop, so we apply formula of drop.

**1.19 15.7**

**Given :**
Diameter of shaft \((d) = 100 \text{ mm} = 0.1 \text{ m}\)
Tangential velocity of shaft \((V) = 10 \text{ m/s}\)
Dynamic viscosity \((\mu) = 0.1 \text{ kg/m.s}\)
Length of shaft \((L) = 100 \text{ mm} = 0.1 \text{ m}\)
Gap between shaft and bearing \((y) = 0.002 \text{ m}\)

Assume velocity gradient is linear, then the shear stress is given by,
\[ \tau = \frac{\mu}{y} \frac{dV}{dy} = \frac{\mu \times V}{y} \]
\[ \tau = 0.1 \times \frac{10}{0.002} = 500 \text{ N/m}^2 \]

Surface area is given by,
\[ A = \pi dL = \pi \times 0.1 \times 0.1 = 0.0314 \text{ m}^2 \]
Frictional resisting force is given by,
\[ F = \tau A \]
\[ F = 500 \times 0.0314 = 15.7 \text{ N} \]
Hence, the frictional resisting force per 100 mm length of the bearing is **15.7 N**.

1.20 **(D)**

Steady, incompressible, laminar flow between one fixed plate and the other moving with a constant velocity.

\[ \tau \]

\[ u(y) \]

\[ y \]

\[ x \]

\[ \text{Stationary plate} \]

\[ U \]

\[ \text{Moving plate} \]

Hence, the correct option is **(D)**.

1.21 **26.4**

**Given**:

Gap between plates \( y = 50 \text{ mm} = 50 \times 10^{-3} \text{ m} \)

Viscosity of the fluid \( (\mu) = 0.44 \text{ kg/m-s} = 0.44 \text{ N-sec/m}^2 \)

Speed of top plate \( (V) = 3 \text{ m/sec} \)

Density of the fluid \( (\rho) = 888 \text{ kg/m}^3 \)

\[ V = 3 \text{ m/sec} \]

\[ h = 50 \text{ mm} \]

\[ \therefore \text{ Assuming linear velocity profile.} \]

According to newton’s law of viscosity,

\[ \tau = \mu \frac{du}{dy} \]

\[ F = \mu \times A \frac{du}{dy} \]

\[ \frac{F}{A} = \mu \frac{du}{dy} \Rightarrow \left( \frac{du}{dy} = \frac{V - 0}{h - 0} \right) \]

\[ \frac{F}{A} = 0.44 \times \frac{3 - 0}{50 \times 10^{-3} - 0} = 26.4 \text{ N/m}^2 \]

Hence, the force per unit area required to maintain the bottom plate stationary is **26.4 N / m}^2**.
According to question, velocity profile is laminar in both the fluids,

\[
\frac{d^2u}{dy^2} = 0, \quad \frac{du}{dy} = c_1 \quad \text{and} \quad u = c_1y + c_2
\]

equation, we can assume linear velocity profile.

If velocity profile is linear, then shear stress will be constant in gap everywhere.

Let \( u \) be the velocity at interface.

At interface, \( \tau_1 = \tau_2 \)

\[
\mu_1 \left( \frac{du}{dy} \right)_1 = \mu_2 \left( \frac{du}{dy} \right)_2
\]

\[
\mu_1 \times \frac{3-u}{2-1} = 2\mu_1 \times \frac{u-0}{1-0}
\]

\[
3 = 2u + u \Rightarrow 3 = 3u
\]

On solving, \( u = 1 \text{ m/s} \)

Hence, the velocity at the interface is \( 1 \text{ m/s} \).

\[\diamondsuit\diamondsuit\diamondsuit\]
Gate Academy Publications introducing the Most Awaited Books for GATE-2020 preparation.

“Inside Knowledge As Precious As Diamond”

- Meticulous Solution of GATE Previous Year Questions (1987-2019)
- Multi Method Approach for a Single Problem to Develop Crystal Clear Concepts
- This book is also value addition for ESE / PSUs / ISRO / DRDO

Booking Starts on 15th April 2019

Sample content also available in Free Download Section of our website

CALL FOR BOOKING : 97131-13156, 0788-4034176